must be the case because both Eqs. (7) and (18) provide the same solution.

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Numerical Analysis of a Finite Wing Altered by a Leading-Edge Ice Accretion

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Introduction

THE disastrous effects of ice accumulation on aircraft are well known. However, for preventative measures to be designed and implemented, the problem must somehow be quantified and various deicing techniques tested. Numerical simulations provide us with a relatively inexpensive way of gathering information on the performance degradation and flowfield characteristics associated with various airfoil/ice accretion shape combinations. Furthermore, they also offer the means by which the effectiveness of deicing methods can be ascertained.

Although research relating to aircraft icing dates back to the 1920s and early 1930s, most modern data have been acquired under a NASA initiative of concurrent experimental and computational research that began in 1978. Specifically, with regard to aircraft performance and flowfield evaluation, the most significant experimental work conducted under this program has been due to Bragg. Beginning his investigations in the early 1980s, Bragg has made use of two distinct shapes known as rime and glaze ice. The former develop at low temperatures and are aerodynamically shaped. Hence, the main consequence of their presence is increased drag due to surface roughness and early boundary-layer transition. On the other hand, glaze accretions, which form at temperatures at or near freezing, are highly irregular and significantly effect the aerodynamics of the flow causing premature stall and excessive drag. Bragg's work spans two decades and includes data on both iced airfoils and iced wings. (see Bragg and Coirier,[1] Bragg and Khodadoust,[2] and Khodadoust and Bragg[3]). It is highly regarded and often used to validate various flow solvers.

Numerical studies have also been performed in the hope that one day highly detailed, reliable information will be available quickly, promoting a thorough analysis of the icing problem and the subsequent development of effective deicing systems. Along this vein there have been two major players to date. The first is Cebeci who used an interactive boundary layer method to evaluate performance losses of a NACA-0012 airfoil with a simulated leading-edge ice shape.[4] Comparison of his results with experimental data indicate good agreement of global properties up to stall. Second, Potapczuk,[5] using the thin-layer Navier–Stokes solver ARC2D, predicted aerodynamic losses beyond stall for the same ice shape and obtained data suggesting that the flow is unsteady with periodic vortex shedding at angles of attack above 7 deg.

Additionally, Kwon and Sankar[6,7] have used a three-dimensional Navier–Stokes code in an effort to analyze an entire wing with leading-edge ice accretion at two different angles of attack. Although the resolution in the boundary layer was quite coarse, the predicted coefficients of lift and pressure were in good agreement with experimental data for an angle of attack of 4 deg. At 8 deg, however, results were poor and prompted further examination. It was then found that modeling of the splitter plate used in the wind tunnel improved results to the point of qualitative agreement.

The present study is a comprehensive effort involving the development and validation of a finite difference-based Navier–Stokes solver and its subsequent use to explore thoroughly the problem of performance degradation due to leading-edge ice accretion. It is a complete study[8] encompassing analysis of both two-dimensional and three-dimensional configurations with the same code. Computations are performed within a three-dimensional parametric space and account for compressibility and variations in Reynolds number. Furthermore, the solver developed is extremely robust. Results obtained from the simulations agree with all previously established experimental and numerical data. Additionally, grid convergence, grid sensitivity, and iteration convergence have been explored guaranteeing that the results obtained are valid and can be reproduced on any appropriate mesh.

In this Note, the results of a specific numerical analysis performed to ascertain the aerodynamics of a wing altered by a leading-edge glaze ice accretion are presented. The wing, made up of NACA-0012 sections, is untwisted and untapered and has an aspect ratio of five. The ice shape employed was the simulated shape used in Refs. 2 and 3, hence facilitating a one-to-one comparison with experimental data and previous computational results.

Computational Methodology

Governing Equations

The governing equations are the complete unsteady, three-dimensional Reynold's-averaged Navier–Stokes equations. To simplify application of the boundary conditions and enhance the overall accuracy of the numerical solution a body-fitted or curvilinear coordinate transformation[9] has been applied to the governing equations. The necessary metrics associated with this approach are computed from the grid using central differences for the interior nodes and second-order one-sided differences at the boundaries.

A two-layer algebraic turbulence model[10] was employed for this research with the eddy viscosity given as

$$\nu_T = l_{\text{mix}}^2|\omega|, \qquad l_i = \kappa y[1 - \exp(-y^+/A)]$$
$$l_o = C_1\delta, \qquad l_{\text{mix}} = \min(l_i, l_o) \tag{1}$$

The constants κ and $A+$ are the von Kármán constant and the van Driest damping coefficient and are taken as 0.41 and 26, respectively. The closure coefficient C_1 is set to 0.089, $|\omega|$ is the magnitude of the vorticity vector, and y^+ is defined as $y(|\tau_w|/\rho_w)^{1/2}/\nu_w$.

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Although more sophisticated turbulence models exist they require much greater computational effort and have not shown any significant advantages when applied to stalled flows.[11] Hence, the use of an algebraic model is not only practical but well justified as evidenced by its widespread use in the literature for aircraft icing problems.

Numerical Algorithm

The algorithm used to solve the governing equations is MacCormack's explicit method.[12] MacCormack's scheme was chosen despite the availability of more modern implicit schemes because it approaches its ultimate solution in a time-accurate manner unhampered by the influence of implicit (second-order) damping. The only term added is fourth-order explicit smoothing necessary to suppress high-frequency oscillations. Because this type of damping is of a higher order than the truncation error associated with the algorithm, the formal accuracy of the numerical algorithm is not compromised by its addition.

To offset the substantial amount of computational effort necessitated by this approach, the code has been fully vectorized. It achieves a sustained performance of 700–800 million floating-point operations on a Cray T90. Furthermore, computational effort is reduced by using an approximate inviscid solution (instead of uniform flow) as the starting point for all computations. This is accomplished with no loss of accuracy because the algorithm used is time accurate and, therefore, will approach the same solution regardless of initial starting point.

A well-posed problem not only requires the application of a good numerical solver but, even more important, complete and accurate specification of appropriate boundary conditions. For the present research, no slip was enforced at the solid surface, which was also assumed to be adiabatic and have a negligible pressure gradient. At the inflow boundaries, located approximately 10 chords from the wing surface in the normal direction and 2.5 chord lengths from wing tip in the spanwise direction, freestream conditions were held fixed. Zero-order extrapolation was used at the outflow 8 chord lengths aft of the trailing edge, and an averaging was used to ensure continuity in the wake region. Last, at the wing root plane, a symmetry condition was imposed.

Results and Discussion

A semispan rectangular wing of aspect ratio five, made of NACA-0012 sections altered by the simulated leading-edge ice accretion used in Refs. 2 and 3, was employed for this research. The mesh on which computations were carried out was formed by linking together a series of C grids created through the use of a hyperbolic grid generator. It contained 177 points in the streamwise direction (107 on the wing itself, 28 defining the ice shape, and 42 in the wake region), 57 points in the normal direction, and 29 points in spanwise direction (19 on the wing). These figures were the result of a careful grid sensitivity analysis[8] and consideration of the cost of the computations. Also, pursuant to the aforementioned numerical analysis, it was determined that a spacing normal to the wing surface of 4.0×10^{-5} chord lengths or less was required to resolve the boundary layer accurately. This is consistent with the spacing of $2.0 \times 10^{-5}c$ used by Potapczuk in a two-dimensional study that utilized the same ice shape.[5]

Global Results

To facilitate comparison with experimental data the simulation was performed at $M_\infty = 0.2$ and $Re_c = 1.5 \times 10^6$. Angles of attack of 4 and 8 deg were analyzed. As Fig. 1 shows, the solver quite accurately predicted the variation of section lift with spanwise distance for both angles of attack. In contrast to previous numerical studies,[6,7] the section lift did not need to be augmented through the use of a solid wall boundary condition designed to compensate for the effects of the splitter plate. The only discrepancy with experimental results,[2,3] was that the simulation slightly delayed the sharp decrease in lift that occurs at outboard stations. Also, due to mesh refinement in the boundary layer, the solver was able to obtain changes in the section drag coefficient. At 4-deg AOA, there

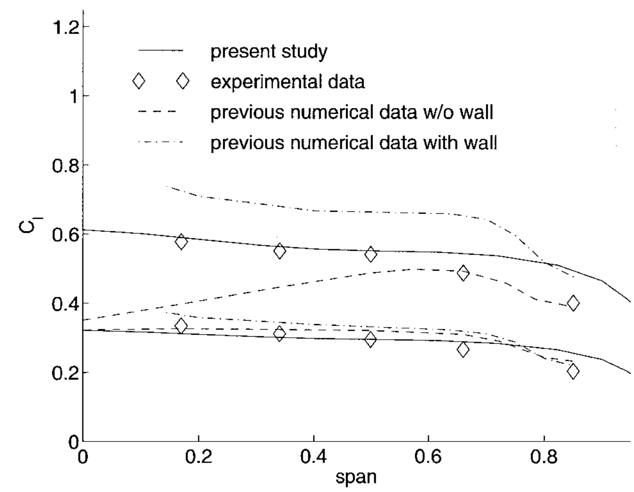

Fig. 1 Coefficient of lift as a function of spanwise location for the iced wing at 4- and 8-deg AOA.

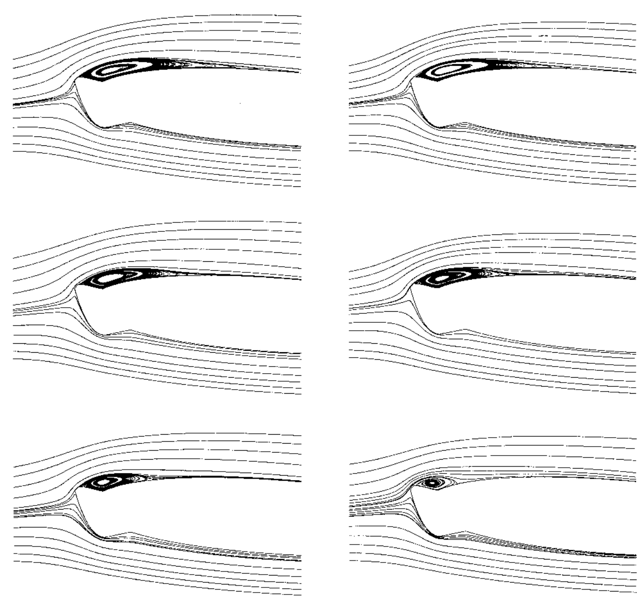

Fig. 2 Streamlines revealing the size and shape of the recirculation bubble at spanwise locations of 0, 41, 72, 82, 94, and 99%, respectively, for a 4-deg AOA.

is little variation (less than 5%) in section drag ($C_d \approx 0.078$) until the actual wing tip is reached. This is because, although the skin friction increases due to the onset of a spanwise flow as outboard stations are approached, the size of the recirculation bubble just aft of the icing horn decreases and causes a corresponding reduction in form drag. At 8-deg AOA, however, the bubble diminishes in size more rapidly. Hence, the overall drag coefficient decreases from a maximum of 0.13 at the wing root plane to a minimum of 0.1 at a location approximately 5% inboard of the wing tip where it begins to rise sharply.

Local Results

At first glance, there appears to be little spanwise variation in the flowfield for the 4-deg case. Closer scrutiny, however, discloses some subtle details. As can be seen in Fig. 2, examination of the streamlines parallel to the wing's cross section reveals that the pressure recovery aft of the separation bubble consistently becomes more pronounced as the wing tip is approached. Initially the tail of the bubble quickly disappears. Although this does not significantly affect the overall size and strength of the bubble, it results in a more sharply defined region of reverse flow. Then, at approximately three-quarters span, the recirculating region begins to dwindle as does the

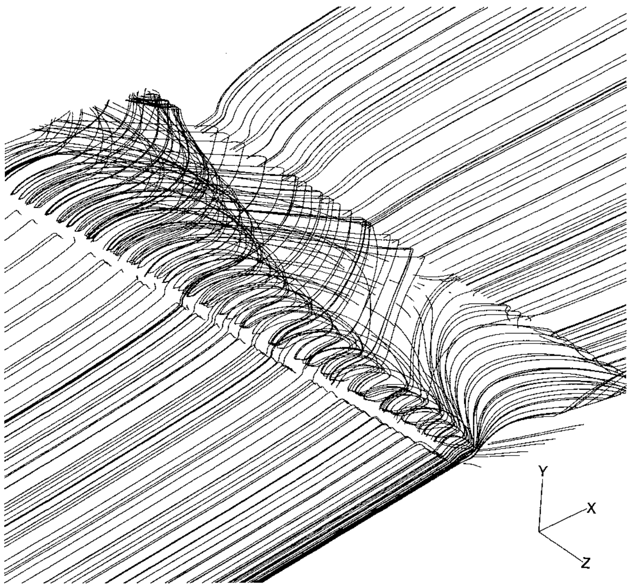

Fig. 3 Streamlines indicating the flow patterns predicted by the simulation at a distance of 0.001 chord lengths normal to the surface for the 8-deg AOA case.

rapid thickening of the boundary layer associated with it. This coincides with the onset of a notable spanwise flow.

Even more interesting is the 8-deg AOA case. As can be seen in Fig. 3, the flow is highly three dimensional in nature and unlike the 4-deg AOA case cannot be construed from two-dimensional data. Indeed the lift coefficient at the centerline is 20% higher than for an infinite wing.[5,8] Moreover, despite the complexity of the flow, the simulation was still able to capture accurately a large amount of detail including variations in surface pressure and boundary-layer thickness. Future analysis of these data and additional poststall results should provide useful insight into the mechanisms of stall for an iced wing. This in turn will help researchers working on the development of ice protection systems focus their efforts in a worthwhile manner.

Conclusions

A numerical investigation into the performance degradation and flowfield evaluation associated with a finite rectangular wing altered by glaze ice has been carried out. Results for the section lift and pressure coefficients at both 4- and 8-deg AOA are in very good quantitative agreement with experimental data. Because of adequate mesh refinement in the boundary layer, it was also possible to obtain accurate data regarding the variation of section drag with spanwise position.

These and other details captured by the simulation indicate that at 4-deg AOA the effects of the spillage of high pressure into low-pressure regions at the wing tip do not unexpectedly alter the flow. The usual decrease in lift as outboard stations are approached occurs, as does a decrease in the size of the separation bubble and a drag rise at the wing tip. In contrast, at 8 deg the flow is inherently three dimensional in nature and cannot be inferred from infinite wing analysis. Hence, more research at angles greater than 8 deg is needed to accurately determine at what point stall occurs for this iced wing and to further explore its poststall behavior.

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